RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.Sc. SECOND SEMESTER (January – June) 2015

Mid-Semester Examination, March 2015

Date : 21/03/2015

MATH FOR ECO (General)

Time : 12 noon – 1 pm

Paper : II

Full Marks : 25

[Use a separate answer book for each group]

Group – A

1. **Answer any two :** [2×2]
   a) State Rolle's theorem. [2]
   b) Prove that $\sin 46^\circ \sim \frac{1}{2}\sqrt{2}\left(1+\frac{\pi}{180}\right)$. [2]
   c) If $y=\cos(m\sin^{-1}x)$ then, find y_n for x = 0. [2]

2. **Answer any one** : [1×3]
   a) Interpret Cauchy's Mean Value Theorem geometrically. [3]
   b) Deduce Lagrange's M.V.T from Rolle's theorem. [3]

3. **Answer any one :** [1×5]
   a) Using M.V.T prove that : $\frac{\pi}{6}+\frac{\sqrt{3}}{15}<\sin^{-1}\left(\frac{3}{5}\right)<\frac{\pi}{6}+\frac{1}{8}$ [5]
   b) If $u_n=D^n(x^n\log x)$, show that $u_n=nu_{n-1}+(n-1)!$.
   Hence, show that $u_n=n!\left(\log x+1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)$. [2+3]

Group – B

4. **Answer any one :** [1×3]
   a) S is the set of all 2×2 real symmetric matrices. Prove that S is a subspace of the vector space $\mathbb{R}_{2\times 2}$ of all 2^{nd} order real matrices. [3]
   b) Let $\{\alpha,\beta,\gamma\}$ be a basis of a real vector space V and c be a nonzero real number. Prove that $\{\alpha+c\beta,\beta,\gamma\}$ is a basis of V. [3]

5. **Answer any two :** [2×5]
   a) Prove that the intersection of two subspaces of a vector space V over a field F is a subspace of V. Is the union of two subspace of V over F is a subspace? Justify your answer. [3+2]
   b) State the Replacement Theorem. Use it to find a basis for the vector space $\mathbb{R}^3$ that contains the vectors (1,2,0) and (1,3,1). [1+4]
   c) Find a basis and dimension of the subspace S of $\mathbb{R}^3$ defined by
   $S=\{(x,y,z)\in\mathbb{R}^3 : x+2y=z, 2x+3z=y\}$. [4+1]

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